353. Proposed by RICHARD P. LOCHNER, Philadelphia, Pa.

The center of a sphere, radius $R = 5$ inches, is $a = 10$ inches above the surface of a sphere, radius $R = 12\frac{1}{2}$ inches. There is a point of light at $b = 1$ inch horizontally from a point $c = 10$ inches vertically above the surface of the first sphere. What is the area of the shadow which the upper sphere casts on the lower one?

354. Proposed by C. N. SCHMALL, New York City.

Prove $\Gamma(1 + x)\Gamma(1 - x) = \dfrac{\pi x}{\sin \pi x}$.

355. Proposed by C. N. SCHMALL, New York City.

Given the curve of the nth degree,

$$y^n - (a + bx)y^{n-1} + (c + dx + ex^2)y^{n-2} + \cdots = 0,$$

show that if each ordinate is divided by the corresponding subtangent, the sum of all the resulting ratios will be a constant.

MECHANICS.

When this issue was made up no solutions had been received for numbers 271, 272, 275, 277, 278, 279, and 282.

286. Proposed by C. N. SCHMALL, New York City.

A slightly elastic string is just long enough to reach between two hooks on the same horizontal line. A ring of weight w is placed at its middle point. Show that the ring will sink through a distance $h = a\sqrt[3]{3ew/2}$, where e is the elasticity of the string and $2a$ the distance between the hooks.

287. Proposed by WALTER H. DRANE, Lebanon, Tenn.

While sitting in an empaled enclosure, I noticed that the spokes of the wheels of passing automobiles, when viewed through the pickets of the fence, appeared to revolve more slowly than they really did, and in some instances even appeared to be revolving in a direction opposite to that in which they were really turning. Explain this optical illusion.

NUMBER THEORY.

When this issue was made up no solutions had been received for numbers 187, 189, 191, 192, 194, 201, and 202 inclusive.

205. Proposed by E. T. BELL, New York City.

Show that in the usual arithmetical sense the form that follows admits of composition; give the requisite transformations, and indicate how several (if not all) solutions may be found. The variables are the x_i.

$$x_0^2 + nrx_1^2 + mrx_2^2 + mnx_3^2 + mnrx_4^2 + mn^2r^2x_5^2 + nr^2m^2x_6^2 + rm^2n^2x_7^2.$$

206. Proposed by R. D. CARMICHAEL, Indiana University.

Prove that the sum of the sixth powers of two integers cannot be the square of an integer.

207. Proposed by A. J. KEMPNER, University of Illinois.

There are 80 positive integers < 100 containing no figure 9 against 19 containing at least one figure 9. (For integers < 1000 the numbers are 728 and 271 respectively.) One might be led to believe that for every positive integer M the number N_1 of positive integers $< M$ containing no figure 9 is always greater than the number N_2 of positive integers $< M$ containing at least one figure 9.

To prove: $\lim_{M=\infty} N_1/N_2 = 0$. See pages 48–50 of January, 1914, issue.